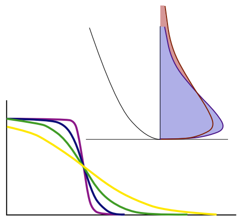

Statistical Mechanics

Homework Assignment 10

Due 6 December 2002

Problem 10.1 – Fluctuations in a Quantum Gas Show for a single orbital that the variance in occupation is given by

$$\left\langle (n - \langle n \rangle)^2 \right\rangle = \langle n \rangle \left(1 \pm \langle n \rangle\right)$$

where the upper sign is for bosons and the lower sign is for fermions.

Problem 10.2 – Average Speed of a Free Electron in Silver The conduction electrons in a noble metal such as silver behave approximately as a free gas of fermions.

(a) Compute the average speed of an electron in terms of the Fermi velocity, which is defined by $\epsilon_{\mathrm{F}} = \frac{1}{2} m v_{\mathrm{F}}^2$.

(b) Assuming that each silver atom in a silver crystal donates one free electron to the conduction band, what is the average speed of the free electrons at room temperature? The density of silver is 10.49 times that of water, the atomic number of silver is $Z = 47$, the atomic mass is 107.9, the rest energy of an electron is 0.511 MeV, and $hc = 1.239$ eV μm.

Problem 10.3 – Fermi-Dirac Integrals In evaluating the shift of the chemical potential for a degenerate fermion gas we encountered the integral

$$I_2 = \int_{-\infty}^{\infty} \frac{x^2\,dx}{(e^x+1)(1+e^{-x})} = \frac{\pi^2}{3} \tag{1}$$

This integral may be evaluated using contour integration for a suitable rectangular path that includes the real axis.

(a) Show that evaluating

$$\oint \frac{z^2\,dz}{(e^z+1)(1+e^{-z})}$$

around a rectangle bounded by the lines at $y = 0$ and $y = 2\pi i$ leads not to the desired integral, but rather to a value for

$$I_0 = \int_{-\infty}^{\infty} \frac{dx}{(e^x+1)(1+e^{-x})} \tag{2}$$

What is that value?

(b) Consider the integral

$$I_\alpha = \oint \frac{e^{\alpha z}\,dz}{(e^z+1)(1+e^{-z})} \tag{3}$$

around the same contour as described in the previous part. Explain how one could obtain both I_0 and I_2 from a closed-form expression for I_α.

(c) Evaluate I_α via contour integration and show that the values you obtain from your expression for I_α for I_0 and I_2 agree with those found and listed above.